Aula 11

Domínios em $\mathbb{R}^2$

- Frações $f(x,y) = \frac{\triangle}{\square}$ $\qquad D_f = \{(x,y) \in \mathbb{R}^2 : \square \neq 0\}$

- Raízes (índice par) $f(x,y) = \sqrt[n]{\square}$ (n par) $\qquad D_f = \{(x,y) \in \mathbb{R}^2 : \square \geq 0\}$

- Logaritmo $f(x,y) = \ln(\square)$ ou $f(x,y) = \log_a(\square)$ $\qquad D_f = \{(x,y) \in \mathbb{R}^2 : \square > 0\}$

- Arco seno ou Arco cosseno $f(x,y) = \arcsin(\square)$ ou $f(x,y) = \arccos(\square)$

$$D_f = \{(x,y) \in \mathbb{R}^2 : -1 \leq \square \leq 1\}$$

Ex 1) a) $f(x,y) = \dfrac{\arcsin(3-x^2-y^2)}{x - \sqrt{y}}$

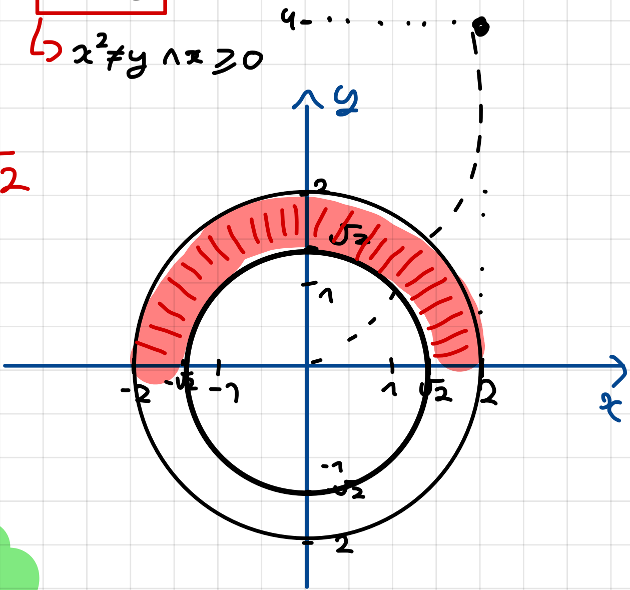
$$D_f = \{(x,y) \in \mathbb{R}^2 : -1 \leq 3-x^2-y^2 \leq 1 \wedge x-\sqrt{y} \neq 0 \wedge y \geq 0\}$$

$$-4 \leq -x^2-y^2 \leq -2 \qquad \boxed{x \neq \sqrt{y}}$$

$\hookrightarrow x^2 \neq y \wedge x \geq 0$

$$4 \geq x^2+y^2 \geq 2$$

raio $\sqrt{4} = 2$ $\qquad$ raio $\sqrt{2}$



b) $f(x,y,z) = \ln(x^2+y^2-z^2-1)$

$$D_f = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2-z^2-1 > 0\}$$

$$x^2+y^2-z^2 > 1$$

exterior de um hiperbolóide de uma folha

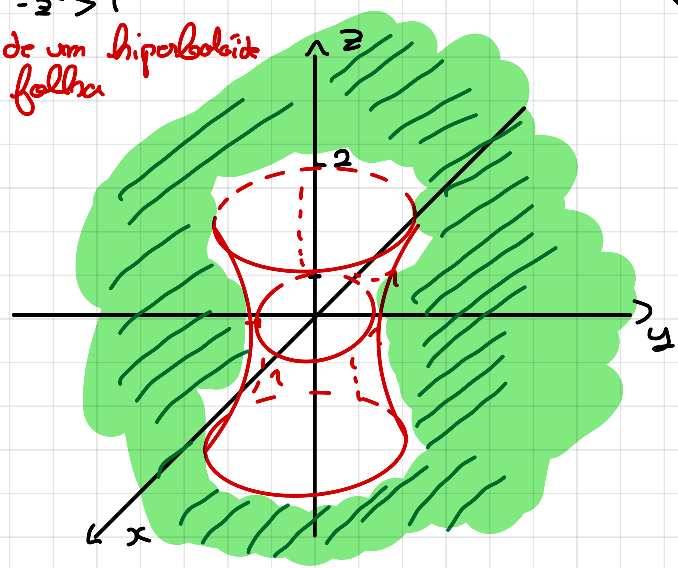
C. aux

$\boxed{z=0}$

$x^2+y^2=1$

$\boxed{z=2}$

$x^2+y^2=5$



2) a) $f(x,y) = x^2+y^2$

$Df = \mathbb{R}^2$

Gráfico de f

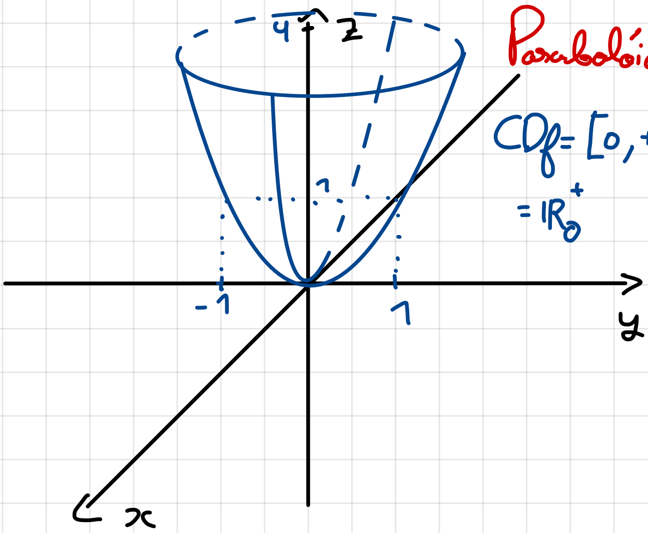
$z = x^2+y^2$

C. aux.

$\boxed{x=0} \to z = y^2$ parabola

$\boxed{y=0} \to z = x^2$ //

$\boxed{z=0} \to x^2+y^2=0 \to$ ponto $(0,0)$



Parabolóide

$CDf = [0,+\infty[$

$= \mathbb{R}_0^+$

$z=4 \rightsquigarrow x^2+y^2=4$

circunferência

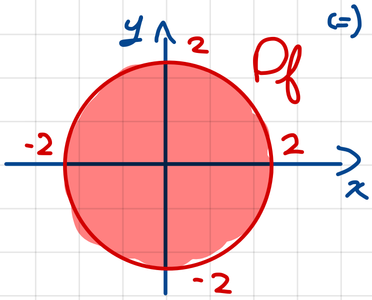
b) $f(x,y) = \sqrt{4-x^2-y^2}$

$Df = \{(x,y) \in \mathbb{R}^2 : 4-x^2-y^2 \geq 0\}$

$\Downarrow$

$-x-y^2 \geq -4$

$\Leftrightarrow x^2+y^2 \leq 4$



Contra-domínio:

como $x^2+y^2 \geq 0$ e pelo Df $x^2+y^2 \leq 4$ então:

$0 \leq x^2+y^2 \leq 4$

$\Leftrightarrow 0 \geq -x^2-y^2 \geq -4$

$\Leftrightarrow 4 \geq 4-x^2-y^2 \geq 0$

$\Leftrightarrow 2 \geq \sqrt{4-x^2-y^2} \geq 0$

$f(x,y)$

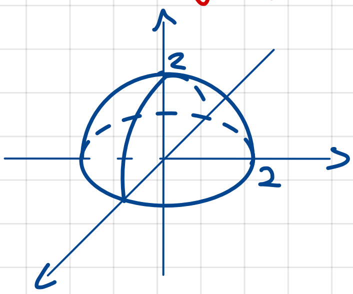
$CDf = [0,4]$

Gráfico de f

$z = \sqrt{4-x^2-y^2}$

$\Leftrightarrow z^2 = 4-x^2-y^2 \quad (z \geq 0)$

$\Leftrightarrow x^2+y^2+z^2=4$

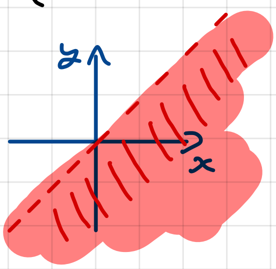


3) a) $f(x,y) = \ln(x-y)$

$Df = \{(x,y) \in \mathbb{R}^2 : x-y>0\}$

$-y > -x$

$y < x$



$z = \ln(x-y)$

$CDf = \mathbb{R}$

b) $f(x,y,z) = \dfrac{1}{x^2+y^2}$

$Df = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \neq 0\} = \mathbb{R}^3 \setminus \{(0,0,z),\ z \in \mathbb{R}\}$

$\Downarrow$

$(x,y) \neq (0,0)$

Todo o $\mathbb{R}^3$ exceto o eixo dos z

$CDf = \mathbb{R}^+$

Mês	Dia
☐	☒
☒	☐
☒	☐
☐	☐
	☒

Intervalo

4) a) $f(x,y) = x^2+y^2$

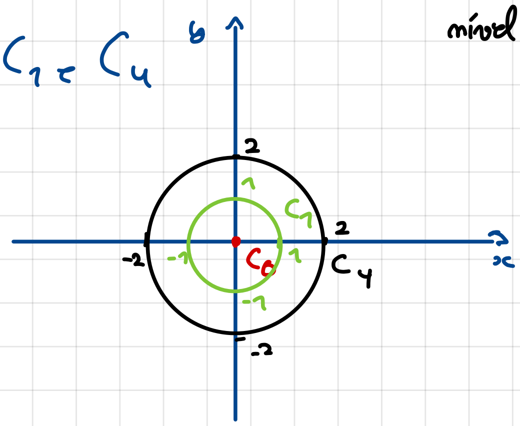
$D_f = \mathbb{R}^2$

$C_K = \{(x,y) \in D_f : f(x,y) = K\}$

$= \{(x,y) \in \mathbb{R}^2 : x^2+y^2 = K\}$

- $K > 0 \rightsquigarrow$ circunferências de centro $(0,0)$ e raio $\sqrt{K}$
- $K = 0 \rightsquigarrow$ ponto $(0,0)$
- $K < 0 \rightsquigarrow$ não existe curva de nível

Esboço de C_0, C_1 e C_4



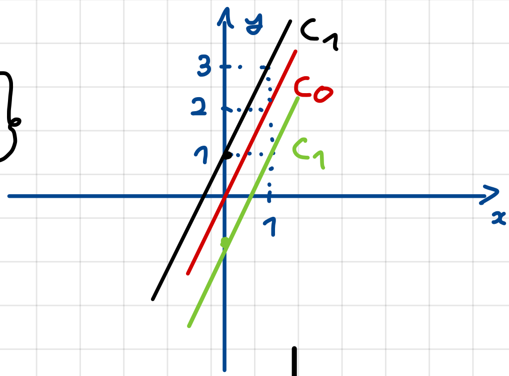
b) $f(x,y) = y-2x$

$D_f = \mathbb{R}^2$

$C_K = \{(x,y) \in \mathbb{R}^2 : y-2x = K\}$

$y = 2x+K$

Retas de declive 2 e ordenada na origem K



4)

c) $f(x,y,z) = x-y^2-3z^2$

$D_f = \mathbb{R}^3$

$S_K = \{(x,y,z) \in \mathbb{R}^3 : x-y^2-3z^2 = K\}$

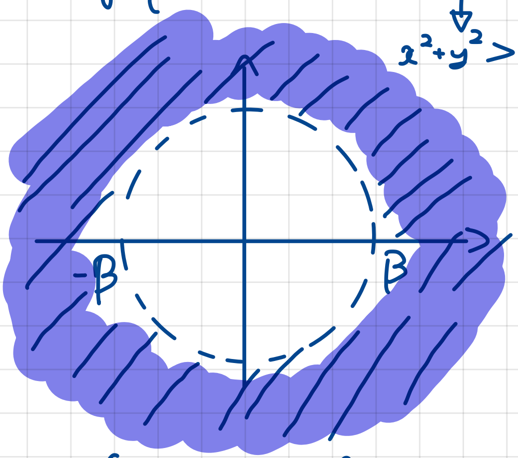
$x = y^2+3z^2+K$

Parabolóide elíptico (no eixo dos "x")

5) $f(x,y) = \ln(x^2+y^2-\beta^2)$, $\beta > 0$

a) $D_f = \{(x,y) \in \mathbb{R}^2 : x^2+y^2-\beta^2 > 0\}$

$x^2+y^2 > \beta^2$



b) $C_K = \{(x,y) \in D_f : \ln(x^2+y^2-\beta^2) = K\}$

$x^2+y^2-\beta^2 = e^K$

$\Leftrightarrow x^2+y^2 = e^K+\beta^2$

Circunferência de centro $(0,0)$ e raio $\sqrt{e^K+\beta^2}$

6) $f(x,y) = \dfrac{x^2 y}{x^4+y^2}$

a)

$D_f = \{(x,y) \in \mathbb{R}^2 : x^4+y^2 \neq 0\}$

$= \mathbb{R}^2 \setminus \{(0,0)\}$

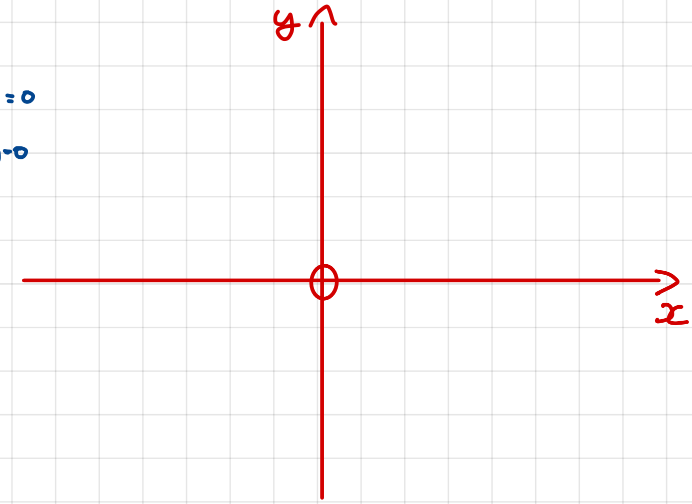
6) $f(x,y)=\dfrac{x^2y}{x^4+y^2}$

b) $C_0 = \left\{(x,y)\in D_f : \dfrac{x^2y}{x^4+y^2}=0\right\} = \left\{(x,y)\in\mathbb{R}^2 : x=0 \vee y=0\right\} \setminus \left\{(0,0)\right\}$

$x^2y=0$

$\Leftrightarrow x^2=0 \vee y=0$

$\Leftrightarrow x=0 \vee y=0$



$C_{1/2} = \left\{(x,y)\in D_f : \dfrac{x^2y}{x^4+y^2}=\dfrac{1}{2}\right\}$

$x^2y = \frac{1}{2}(x^4+y^2)$

$\Leftrightarrow 2x^2y = x^4+y^2$

$\Leftrightarrow x^4+y^2-2x^2y=0$

$\Leftrightarrow (x^2-y)^2=0$

$\Leftrightarrow x^2-y=0$

$\Leftrightarrow y=x^2$